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Statistical analysis of mineral diversity and distribution: Earth's mineralogy is unique

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ABSTRACT

Earth's mineralogical diversity arises from both deterministic processes and frozen accidents. We apply statistical methods and comprehensive mineralogical databases to investigate chance versus necessity in mineral diversity-distribution relationships. Hundreds of mineral species, including most common rock-forming minerals, distinguish an "Earth-like" planet from other terrestrial bodies. However, most of Earth's ∼5000 mineral species are rare, known from only a few localities. We demonstrate that, in spite of deterministic physical, chemical, and biological factors that control most of our planet's mineral diversity, Earth's mineralogy is unique in the cosmos.

1. Introduction

What constitutes an "Earth-like" planet? In the search for exoplanets, mass and radius (and thus density) constitute first-order astronomical determinants of Earth-like worlds. However, many terrestrial planets will not possess key traits of Earth, such as a hydrosphere and plate tectonics. Consequently, a compelling definition of Earth-like planets remains elusive (Brownlee and Ward, 2004; Segura et al., 2005; Sotin et al., 2007; Svedhem et al., 2007; Kaltenegger and Traub, 2009). We suggest that mineralogical criteria—specifically the diversity and distribution of near-surface mineral species—provide robust indicators of geochemical and tectonic environments that influence the evolution of a terrestrial planet, including its oceans, atmosphere, and life.

Complex systems such as minerals, organic molecules, and life evolve in both deterministic and stochastic ways (Monod, 1971; Hazen and Eldredge, 2010; Lecca et al., 2013; Davila and McKay, 2014). Many aspects of these complex systems are inevitable consequences of their initial conditions and the subsequent influence of physical and chemical laws. However, complex systems also display frozen accidents that are integral to evolutionary pathways. A tension thus exists between chance and necessity—a tension that is heightened because in most natural systems no quantitatively rigorous methods exist to determine which is which. In such cases, distinguishing chance from necessity becomes more a philosophical debate than a scientific pursuit (Gould, 2002; Conway-Morris, 2003; Pearce, 2010).

Hazen and coworkers have demonstrated that the diversity and distribution of minerals on Earth have evolved over more than 4.5 billion years through a combination of physical, chemical, and biological processes (Hazen et al., 2008, 2011; Grew and Hazen, 2014), and they have explored quantitatively the roles played by both chance and necessity in observed mineral diversity/distribution systematics (Hazen et al., in press (a)). In contrast to many other complex natural evolving systems, minerals are documented with comprehensive species/locality data, notably the official International Mineralogical Association compilation of 4831 approved mineral species, rruff.info/ima (as of 1 February 2014; Downs, 2006), and 652,865 unique data of a mineral species at a specific locality (hereafter referred to as "species/locality data") recorded by the crowd-sourced website mindat.org (also as of 1 February 2014). Hystad et al. (in press) demonstrated that these data conform to Large Number of Rare Events (LNRE) frequency distribution functions, which permit evaluation of probabilities for the occurrence of mineral species on other Earth-like planets, or if a hypothetical "tape of Earth history" were replayed.

In order to evaluate deterministic versus stochastic aspects of Earth's near-surface mineralogical environment, we focus on

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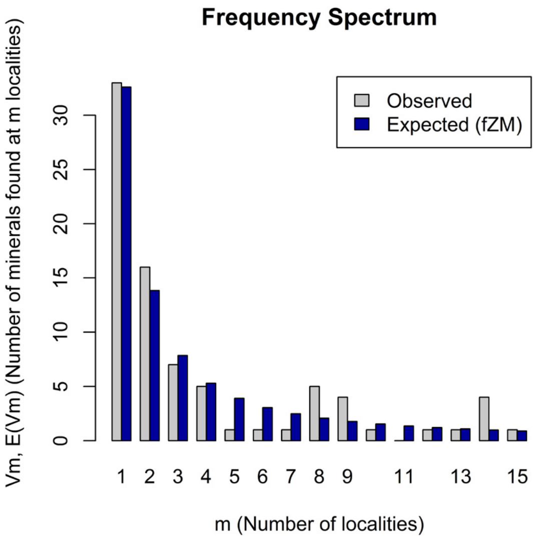

Fig. 1. The number of Be mineral species (y axis) versus the number of reported localities ($m = 1$ to 15 localities, based on data in mindat.org as of 1 February 2014). The observed (light) and modeled (dark) frequency spectrum of Be minerals fits a finite Zipf–Mandelbrot (*fZM*) Large Number of Rare Events (LNRE) distribution. Thus, half (56) of the 112 known Be mineral species (rruff.info/ima as of 1 February 2014) are known from 3 or fewer localities, whereas only 10 species are known from more than 100 localities.

the tractable case of the 112 approved mineral species in which the rare element Be is essential (rruff.info/ima; Grew and Hazen, 2014), as documented from 6778 Be species/locality data (mindat.org). This well-documented subset of the 4831 mineral species approved by the International Mineralogical Association lends itself to quantitative statistical analysis because Be minerals display an LNRE frequency distribution similar to that observed for all mineral species (Hystad et al., in press). More than 50% of all recorded species/locality data are associated with the commonest species, beryl (ideally $\mathrm{Be_3Al_2Si_3O_{12}}$, with 3415 reported localities), whereas half of all Be species are rare (i.e., 56 species have been reported from three or fewer localities). Such LNRE frequency distributions are also characteristic of biomass in a forest ecosystem or words in a book (Baayen, 2001; Evert and Baron, 2008). In particular, Be mineral frequency distribution follows a finite Zipf–Mandelbrot LNRE distribution or finite Zipf–Mandelbrot model formulated as an LNRE model (fZM), as illustrated in Fig. 1 (Evert, 2004; Hystad et al., in press). We fit the fZM LNRE model's 3 parameters to the beryllium mineral frequency spectrum using the R-package, zipfR (Evert and Baroni, 2007; Hazen et al., in press (b)). We estimated parameters by minimization through a custom estimation procedure, the simplified version of the multivariate chi-squared test for goodness-of-fit using the first 10 spectrum elements.

Our parameters are $\alpha = 0.311$, $A = 1.264 \times 10^{-5}$, and $B = 0.217$; consequently, $C = 1.978$, with $\chi^2 = 3.05$, $\mathrm{d}f = 3$, and p-value $= 0.38$ (see Supplementary Materials).

The conformity of Be minerals to a fZM distribution is convenient because probabilities of mineral occurrence, including prediction of the number of species that exist but have not yet been discovered, can be calculated. Fig. 2 illustrates the Be mineral species accumulation curve—a monotonically increasing relationship between the total number of reported mineral species/locality data for Be minerals (x axis) and the number of described Be mineral species (y axis). For example, as of 1 February 2014, 112 Be mineral species had been reported from 6778 localities. Because Be minerals conform to a fZM distribution, the total number of known Be mineral species should increase at a predictable rate as the reported number of localities for Be minerals increases. Thus, the fZM formalism leads to an accumulation curve that predicts the existence of 91 additional beryllium mineral species on Earth that have yet to be documented, in addition to the 112 known species—an estimated total of 203 Be minerals on Earth today. Geochemical and crystal chemical analyses suggest that many of the 91 “missing” Be minerals could be natural analogs of known synthetic compounds or isomorphic compositional variants of known Be minerals (Grew and Hazen, 2014). Note that similar statistical methods can be applied to other subsets of minerals, as well as to all known species (Hazen et al., in press (a), in press (b)).

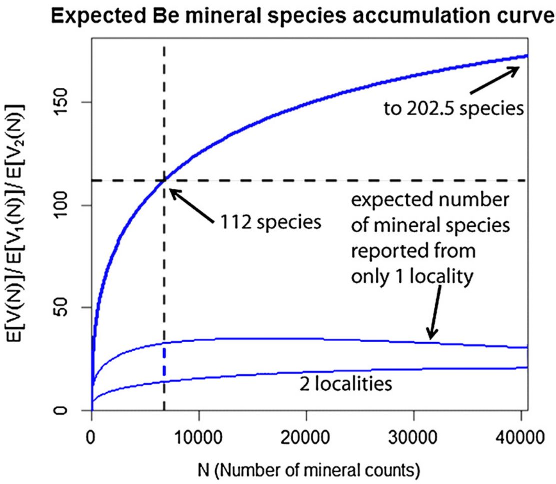

Fig. 2. Expected Be mineral species accumulation curves, extrapolated at 6 times the sample size ($N = 6778$), using Sichel's model. The upper curve indicates the expected number of distinct mineral species, $E(V(N))$, versus the sample size, N. The point at which the vertical dashed line intersects the x-axis denotes the current value of the sample size $N = 6778$, for which the current number of known Be mineral species $E(V(N)) = 112$. Extrapolation of the upper accumulation curve suggests that 91 Be minerals have yet to be discovered. The lower two accumulation curves represent the numbers of mineral species found at exactly 1 and 2 localities, respectively.

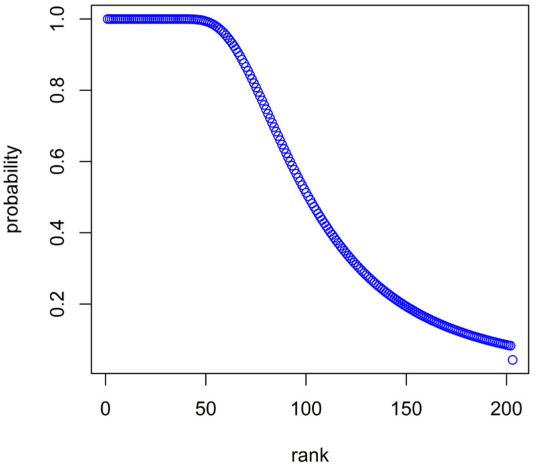

Fig. 3. Rank versus probability that a Be mineral species in the population of 203 Be species will be present at least one time in a sample with $N = 6778$, where N is the number of Be mineral species-locality data (see also Supplementary Table 1).

These statistical data facilitate estimates of the probable mineralogical similarities and differences in two Earth-like planets. Accordingly, we calculated the probabilities of occurrence of 203 beryllium mineral species for Earth (Fig. 3; Supplementary Table 1). We find that 34 mineral species are likely to occur on virtually all Earth-like planets, while an additional 26 mineral species have >95% probability of occurrence. These 60 most common beryllium mineral species—54% of the presently known Be mineral diversity—thus represent the suite of Be minerals that are most likely to exist on Earth today and will likely exist on another Earth-like planet, as well.

By contrast, we find that the probabilities of occurrence for 102 rare species—half of those predicted to occur on Earth today—are $<50\%$, while the 14 least likely mineral species have probabilities $<10\%$ (Fig. 3; Supplementary Table 1). These species individually, and especially in combination, represent mineral distributions that occur by chance.

The values in Supplementary Table 1 represent the probabilities that each of the Be mineral species in the population will occur at least once on a randomly selected Earth-like planet on which 6778 mineral species/locality data have been documented. To obtain an upper bound for the probability of discovering the same 112 Be mineral species on another Earth-like planet we multiply the probabilities for the first 112 ranked mineral species in the population (i.e., the most probable species as indicated by the first 112 values in Supplementary Table 1). The resulting probability that the identical 112 most probable Be mineral species would be found on another Earth-like planet is thus less than 1 in 1.35×10^{-10}.

Note that the actual probability for the distribution of Be mineral species on another Earth-like planet will be significantly smaller than this estimate for at least two reasons. First, this calculation does not take into account hundreds of plausible Be mineral species that are not present on Earth today, but could occur on many other Earth-like planets. Hazen et al. (in press (a)) estimated that every mineral-forming chemical element has the potential to form at least 1000 species; thus, we suggest that as many as 800 additional plausible Be minerals may occur in the cosmos, significantly decreasing the likelihood of finding the identical suite of 112 species on another planet. Second, this calculation assumes a planet with identical compositional and other initial conditions to Earth. However, recent observations of varied stellar stoichiometries (Chambers, 2010; Nissen, 2013; Young et al., 2014), planetary orbits (Borucki et al., 2011; Kopparapu, 2013; Hadden and Lithwick, 2014), and mass-to-radius ratios (Seager et al., 2007; Kaltenegger et al., 2013), not to mention inevitable differences in another planet's biosphere (should there be one), suggest that Earth-like planets may be rare (Brownlee and Ward, 2004; Hazen et al., in press (a)). Thus, we conclude that significantly less than 1 terrestrial planet in 10^{10} could possess the same suite of Be mineral species.

The preceding calculations relate specifically to the distribution of 112 known beryllium minerals, which represent only 2.3% of the 4831 approved species. Given Earth's significant mineralogical diversity, how probable is Earth's mineralogy on another Earth-like planet? The most common rock-forming minerals are likely to dominate on any Earth-like planet; thus, major lithological units such as basalt and granite are deterministic. However, the majority of mineral species are rare, with 1062 species known from only 1 locality (mindat.org). Thus, replicating Earth's documented mineralogy on another world that has been subjected to comparable exploration would require more than 1000 low-probability (less than 50%) events. Extrapolating from the case of Be minerals, for which 33 species are known from 1 locality, and assuming that probabilities scale roughly with the number of rare minerals (also noting that similar LNRE frequency distributions apply both to all minerals and to the Be subset), we approximate the probability, P, of duplicating Earth's mineralogy to be less than:

$$P < 10^{-[10\times(1062/33)]} \approx 10^{-322}$$

Estimates based on dark-field imaging suggest that the universe holds approximately 10^{11} galaxies (Williams et al., 1996), each with approximately 10^{11} stars. If every star on average is orbited by one terrestrial planet or moon, there may be 10^{22} terrestrial (if not "Earth-like") worlds. Given the significant estimated mineralogical differences between any two identical Earth-like planets, amplified by the likelihood that no two planets are identical in all aspects of their initial chemical and physical parameters, we conclude that Earth's mineralogy is unique in the cosmos.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.epsl.2015.06.028.

References

Baayen, R.H., 2001. Word Frequency Distributions. Kluwer, Dordrecht, The Netherlands.

Borucki, W.J., Koch, D.G., Basri, G., et al., 2011. Characteristics of Kepler planetary candidates based on the first data set. Astrophys. J. 728 (2), 117–136.

Brownlee, D., Ward, P., 2004. Rare Earth: Why Complex Life is Uncommon in the Universe. Copernicus, New York.

Chambers, J., 2010. Stellar elemental abundance patterns: implications for planet formation. Astrophys. J. 724, 92–97.

Conway-Morris, S., 2003. Life's Solution: Inevitable Humans in a Lonely Universe. Cambridge University Press, Cambridge, UK.

Davila, A.F., McKay, C.P., 2014. Chance and necessity in biochemistry: implications for the search for extraterrestrial biomarkers in Earth-like environments. Astrobiology 14, 534–540.

Downs, R.T., 2006. The RRUFF project: an integrated study of the chemistry, crystallography, Raman and infrared spectroscopy of minerals. In: Program Abstracts 19th General Meeting Int. Mineral. Assoc., Kobe, Japan. O03-13.

Evert, S., 2004. Simple LNRE model for random character sequences. In: Proc. JADT, pp. 411–422.

Evert, S. Baron, M., 2008. Statistical Models for Word Frequency Distributions. Package zipfR.

Evert, S., Baroni, M., 2007. zipfR: word frequency distributions in. In: R. Proc. Annual Meeting Assoc. Computational Linguistics, Posters and Demonstrations Session, vol. 45, pp. 29–32.

Gould, S.J., 2002. The Structure of Evolutionary Theory. Belknap Press, Cambridge, MA.

Grew, E.S., Hazen, R.M., 2014. Beryllium mineral evolution. Am. Mineral. 99, 999–1021.

Hadden, S., Lithwick, Y., 2014. Densities and eccentricities of 139 Kepler planets from transit time variations. Astrophys. J. 787 (1), 80.

Hazen, R.M., Eldredge, N.L., 2010. Themes and variations in complex systems. Elements 6, 43–46.

Hazen, R.M., Papineau, D., Bleeker, W., Downs, R.T., Ferry, J., McCoy, T.L., Sverjensky, D.A., Yang, H., 2008. Mineral evolution. Am. Mineral. 93, 1693–1720.

Hazen, R.M., Bekker, A., Bish, D.L., Bleeker, W., Downs, R.T., Farquhar, J., Ferry, J.M., Grew, E.S., Knoll, A.H., Papineau, D., Ralph, J., Sverjensky, D.A., Valley, J.W., 2011. Needs and opportunities in mineral evolution research. Am. Mineral. 96, 953–963.

Hazen, R.M., Grew, E.S., Downs, R.T., Golden, J., Hystad, G., in press (a). Mineral ecology: chance and necessity in the diversity of terrestrial planets. Can. Mineral. http:dx.doi.org/10.3749/canmin.1400086.

Hazen, R.M., Hystad, G., Downs, R.T., Golden, J., Pires, A.J., Grew, E.S., in press (b). Earth's "missing" minerals. Am. Mineral. http://dx.doi.org/10.2138/am-2015-5417.

Hystad, G., Downs, R.T., Hazen, R.M., in press. Mineral species frequency distribution conforms to a large number of rare events model: prediction of Earth's missing minerals. Math. Geosci.. http://dx.doi.org/10.1007/s11004-015-9600-3.

Kaltenegger, L., Traub, W.A., 2009. Transits of Earth-like planets. Astrophys. J. 698 (1), 519–527.

Kaltenegger, L., Sasselov, D., Rugheimer, S., 2013. Water-planets in the habitable zone: atmospheric chemistry, observable features, and the case of Kepler-62e and -62f. Astrophys. J. Lett. 775, 47–52.

Kopparapu, R.K., 2013. A revised estimate of the occurrence rate of terrestrial planets in the habitable zones around Kepler *m*-dwarfs. Astrophys. J. Lett. 767, L8.

Lecca, P., Laurenzi, I., Jordan, F., 2013. Deterministic Versus Stochastic Modeling in Biochemistry and Systems Biology. Woodhouse Publishing, Cambridge, UK.

Monod, J., 1971. Chance and Necessity: Essay on the Natural Philosophy of Modern Biology. Knopf, New York. Translated from the French by A. Wainhouse.

Nissen, P.E., 2013. The carbon to oxygen ratio in stars with planets. Astron. Astrophys. 552, 73–83.

Pearce, T., 2010. Convergence and parallelism in evolution: a neo-Gouldian account. Br. J. Philos. Sci. 63, 429–448.

Seager, S., Kuchner, M., Hier-Majumder, C.A., Militzer, B., 2007. Mass-radius relationships for solid exoplanets. Astrophys. J. 669, 1279–1297.

Segura, A., Krelove, K., Kasting, J.F., Sommerlatt, D., Meadows, V., Crisp, D., Cohen, M., Mlawer, E., 2005. Ozone concentrations and ultraviolet fluxes on Earth-like planets around other stars. Astrobiology 3 (4), 689–708.

Sotin, C., Grasset, O., Mocquet, A., 2007. Mass-radius curve for extrasolar Earth-like planets and ocean planets. Icarus 191 (1), 337–351.

Svedhem, H., Titov, D.V., Taylor, F.W., Witasse, O., 2007. Venus as a more Earth-like planet. Nature 450, 629–632.

Williams, R.E., HDF Team, 1996. The Hubble deep field: observations, data reduction, and galaxy photometry. Astron. J. 112, 1335. http://dx.doi.org/10.1086/118105.

Young, P.A., Desch, S.J., Anbar, A.D., Barnes, R., Hinkel, N.R., Kopparapu, R., Madhusudhan, N., Monga, N., Pagano, M.D., Riner, M.A., Scannapieco, E., Shim, S.-H., Truitt, A., 2014. Astrobiological stoichiometry. Astrobiology 14, 603–626.